

Condensate localization in a quasi-periodic structure

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Abstract

We propose a set-up of optical laser beams by which one may realize a quasi-one-dimensional Fibonacci array of potential wells for a Bose-Einstein condensate. We use a Bose-Hubbard tight-binding model to evaluate the transport of superfluid ^{87}Rb atoms driven by a constant force through such an array. We show that the minibands that are generated in the spectral density-of-states by the quasi-periodic disorder give rise to prominent localization effects, which can be observed by measuring the tunnel output of matter into vacuum as a function of the intensity of the applied force.

Key words: Bose-Einstein condensates, transport properties, optical laser applications

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1 Introduction

It is well known from solid-state physics that the introduction of quasi-periodic disorder in a system of particles on a lattice profoundly modifies the single-particle spectral density of states (DOS) and induces localization at certain energies. Here we discuss a way in which these effects are manifested in a superfluid Bose-Einstein condensate and how they may be experimentally demonstrated.

In more detail, we use a Bose-Hubbard tight-binding model for a superfluid assembly of ^{87}Rb atoms to calculate the transport of matter driven by a constant force through a quasi-one-dimensional (1D) array of potential wells obeying the Fibonacci sequence. Parallel calculations are carried out for the current flowing under the same conditions through a perfectly periodic array. As in our previous studies of transport in a condensate [1,2], our calculations use a Green's function method in which the array is reduced by a renormalization/decimation technique to a single dimer connected to incoming and outgoing leads [3]. The current is assessed from the output of matter tunnelling into vacuum. We show that the minigaps that are generated in the DOS by quasi-periodicity give rise to prominent localization effects, which are revealed in the output current as a function of the intensity of the applied force.

We then propose a method by which one may realize in the laboratory a quasi-one-dimensional array of potential wells obeying the Fibonacci sequence for a Bose-Einstein condensate. We use the idea that a quasi-periodic array of dimensionality d can be created by suitably projecting a periodic array of dimensionality $2d$ onto a space of dimensionality d [4]. In the case of present interest, we propose a set-up of four optical laser beams to create a square lattice and a suitably directed hollow beam to confine the atoms in a strip which is oriented relative to the lattice according to the golden ratio. Gravity acting on a tilted assembly may be used to generate a constant drive.

2 The model and its numerical solution

Our theoretical approach has been described in detail in our previous studies [1,2] and here we report only its essential points. The 1D Bose-Hubbard Hamiltonian for N bosons distributed inside n_s potential wells is

$$H_I = \sum_{i=1}^{n_s} [E_i |i\rangle\langle i| + \gamma_i (|i\rangle\langle i+1| + |i+1\rangle\langle i|)] . \quad (1)$$

where E_i and γ_i are site energies and hopping energies, respectively. In a tight-binding scheme the condensate wave function $\phi_i(z)$ in a quasi-1D harmonic

well can be represented by a Gaussian Wannier function of axial width σ_i [5]. The parameters entering the effective Hamiltonian are then given by

$$E_i = \int dz \phi_i(z) \left[-\frac{\hbar^2 \nabla^2}{2m} + U_i(z) + \frac{1}{2} g_b |\phi_i(z)|^2 - Fz + \mathcal{C} \right] \phi_i(z) \quad (2)$$

and

$$\gamma_i = \int dz \phi_i(z) \left[-\frac{\hbar^2 \nabla^2}{2m} + U_i(z) + \frac{1}{2} g_b |\phi_i(z)|^2 + \mathcal{C} \right] \phi_{i+1}(z). \quad (3)$$

In Eqs. (2) and (3) U_i is the external potential acting on the bosons in the i -th well, F is a constant external force, g_b is the effective 1D boson-boson coupling strength, and \mathcal{C} is a constant accounting for transverse confinement. Nonlinear interaction effects enter the self-consistent determination of the widths σ_i , so that the condensate wave function $\phi_i(z)$ and the parameters in the Hamiltonian H_I also depend on the number of bosons in each well [1]. This approach is well justified in the case of weak boson-boson coupling as for a ^{87}Rb condensate, which we study in this work.

In the Green's function method the calculation of transport by bosonic matter waves through the array of potential wells does not require an explicit solution of the Hamiltonian (1) [3]. As already noted, the array is reduced to a single dimer to which an incoming lead and an outgoing lead are connected. The incoming lead injects particles in the zero-momentum state and the outgoing lead, in the case of a perfectly periodic array, extracts them by allowing tunnel into vacuum after acceleration by the constant external force up to the edge of the Brillouin zone. Since the presence of the constant force tilts the array (cf. Eq. (2)), the outgoing lead has to be connected to a well whose position is determined by the magnitude of F according to the above criterion. The position of the outgoing lead for each value of F is preserved in going from a periodic to a Fibonacci array. The steady-state transport coefficient is then inferred from the scattered wave function of the leads in the presence of the effective dimer.

3 Density of states

In order to understand the results for the transport coefficient in the two cases of a periodic and a quasi-periodic array, it is useful to exhibit first the DOS for the two cases. Its calculation has been carried out by recursive algorithms such as those presented in Refs. [6,7].

The total DOS at energy E is defined as

$$n(E) = -\frac{1}{\pi} \text{Im} \sum_i^{n_s} \langle i | \hat{G}(E) | i \rangle, \quad (4)$$

where $\hat{G}(E)$ is the single-particle Green's function operator. Figure 1 reports $n(E)$ as a function of energy over the whole energy band for a very long periodic array (1000 sites) and for the corresponding Fibonacci array. The latter has been generated by arranging two types of sites with energies E_1 and E_2 in a sequence determined according to the Fibonacci chain rule $ABBABABB\dots$. The sequence is generated by the transformation rule $A \rightarrow B$ and $B \rightarrow BA$.

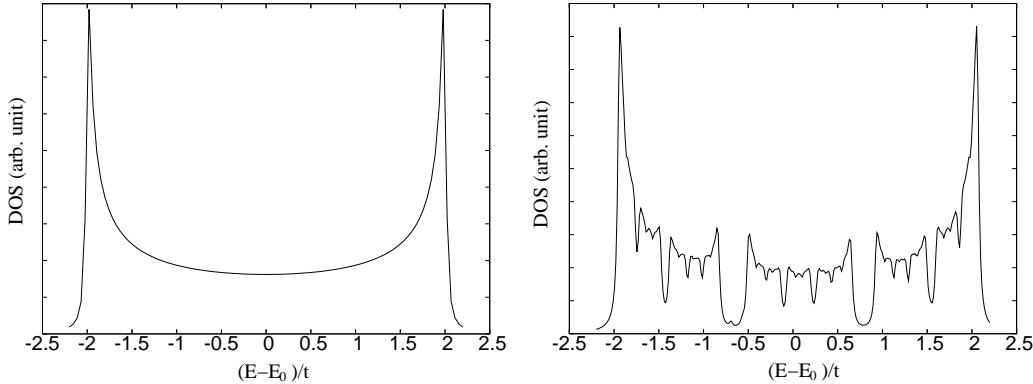


Fig. 1. Total density of states DOS of a periodic array of potential wells (left panel) and of a quasi-periodic Fibonacci array (right panel), as a function of the energy E referred to the band centre at energy E_0 , for a total band width equal to $4t$.

It is evident from Fig. 1 that the introduction of quasi-periodicity leads to a fragmentation of the spectrum of single-particle energies. This is a typical product of introducing disorder in a periodic system by quasi-periodic or even aperiodic modulations of the site energies. In particular, in the classical case of a quasi-periodic Fibonacci chain the spectrum is known to be a Cantor set with zero measure. The emergence of minigaps in the DOS causes the effects of particle localization that we shall illustrate in the next section.

Figure 2 presents a comparison between the site-projected DOS of a periodic and a Fibonacci array of 100 wells under the action of gravity. This quantity is defined as

$$n_i(E) = -\frac{1}{\pi} \text{Im} \langle i | \hat{G}(E) | i \rangle, \quad (5)$$

and is shown in Fig. 2 at the energy $E = E_0 - 2t$, that is at the bottom of the energy band. The site-projected DOS in the Fibonacci chain has a general envelope resembling that of the periodic chain, but is modified by the quasi-periodicity favouring the population of a subset of sites.

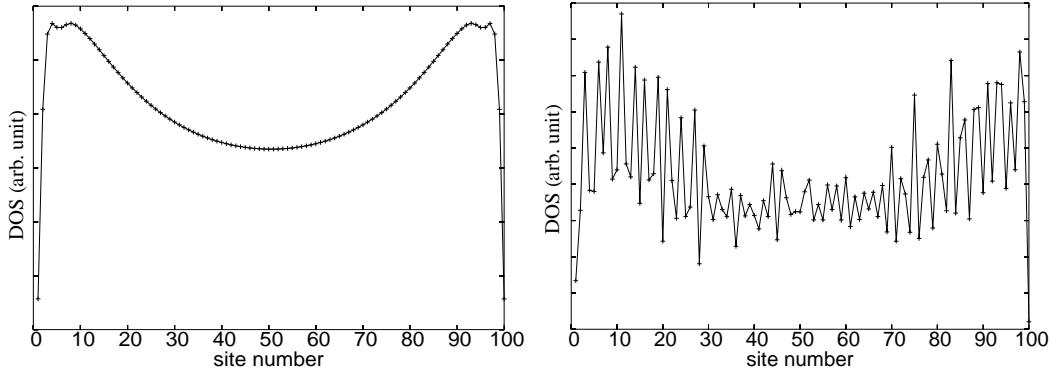


Fig. 2. Projected DOS of a periodic array (left panel) and of a Fibonacci array (right panel), as a function of the site number. Both arrays extend over 100 sites.

4 Localization from quasi-periodicity

The transmittivity coefficient \mathcal{T} of condensate matter through the quasi-1D array is determined by the Green's function element describing the coherence between the input site and the output site (see Ref. [1] for a full discussion of the treatment and of the computational method). This coefficient depends on the magnitude of F for each given value of the energy difference $\Delta E = |E_1 - E_2|$. Figure 3 reports our results for the case of a periodic array ($\Delta E = 0$) and for three Fibonacci arrays constructed with increasing values of the ratio $\Delta E/\bar{E} = 2|E_1 - E_2|/|E_1 + E_2|$. The transmittivity is plotted in Fig. 3 as a function of the magnitude of the force F , expressed either through the ratio of the acceleration $a = F/m$ to the acceleration of gravity g or through its inverse, that is the ratio of the corresponding periods of Bloch oscillations $T_B/T_{B_g} = g/a$. Notice that the lines drawn in Fig. 3 serve the only purpose of guiding the eye from each data point to the next.

It is seen from Fig. 3 that, while in the case of the periodic array the transmittivity coefficient is a smoothly increasing (nonlinear) function of the drive, the introduction of even a modest amount of quasi-periodic disorder (for $\Delta E/\bar{E} = 3 \times 10^{-3}$, in the third panel from the top) suffices to introduce a great deal of structure. In fact, the transmittivity coefficient is directly proportional to the outgoing current, since the difference in chemical potential between the two leads is fixed by the total band-width and does not depend on the strength of the drive [1]. Thus the low values of \mathcal{T} in the third panel in Fig. 3 directly reflect low values of the particle current due to scattering against the quasi-periodic disorder. The minima effectively become zeroes in the bottom panel, corresponding essentially to localization of the atoms in a finite array with quasi-periodic disorder of 1% magnitude.

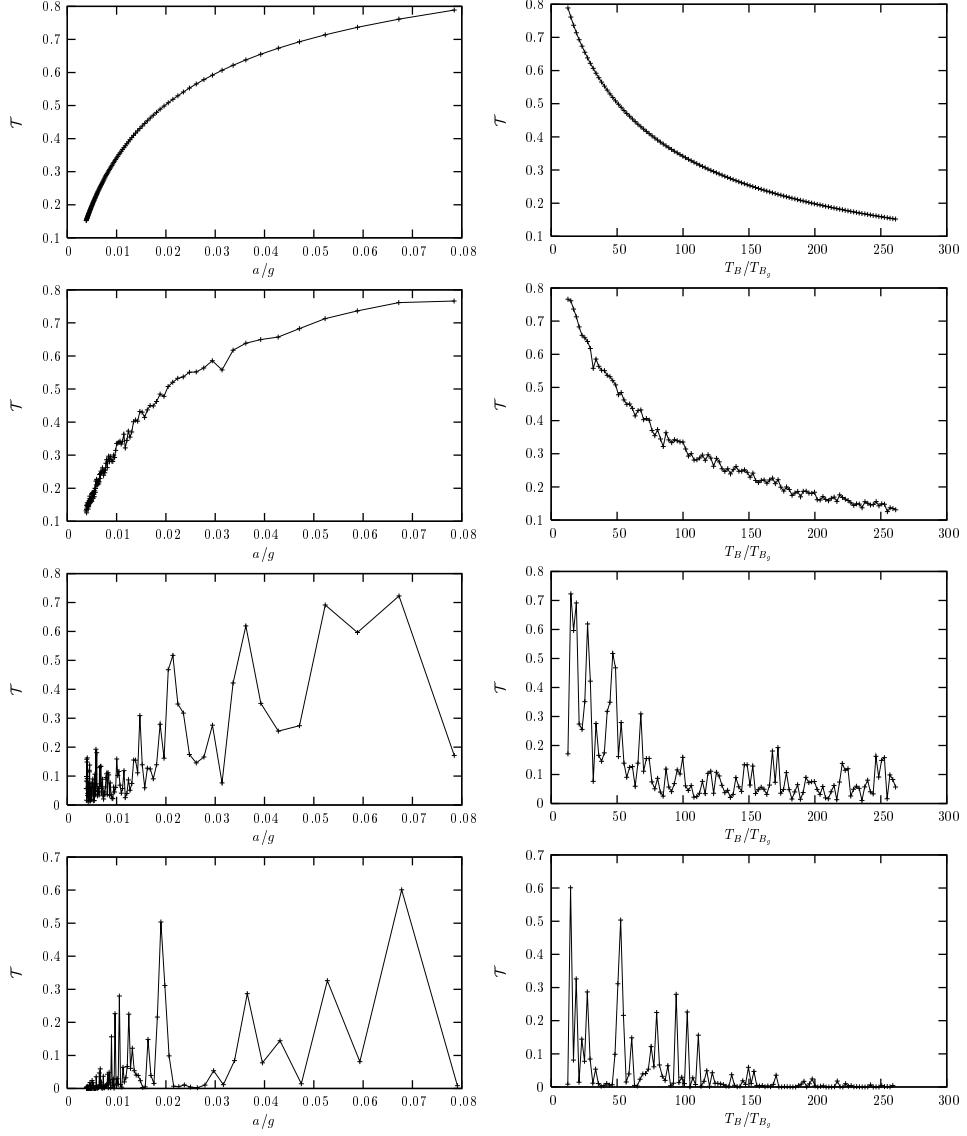


Fig. 3. Condensate transmittivity as a function of its acceleration $a = F/m$ (in units of the acceleration of gravity g , first column) and of its inverse $g/a = T_B/T_{B_0}$ for a periodic array (first row) and for Fibonacci arrays with increasing quasi-periodic disorder. The second, third, and fourth row correspond to $\Delta E/\bar{E} = 3 \times 10^{-5}$, 3×10^{-3} , and 10^{-2} , respectively.

5 Proposed set-up for an atomic Fibonacci waveguide

Although our numerical illustration has referred to the situation in which the quasi-periodic disorder is created by modifying the site energies, it may be easier to realize experimentally a set-up in which the disorder is introduced in the hopping energies. The behaviour of matter waves propagating through

such a quasi-periodic array should not differ qualitatively from that illustrated in Sec. 4 above.

A schematic drawing of the set-up of optical lasers that would create an atomic Fibonacci wave guide is shown in Fig. 4. Here, two pairs of counter-propagating laser beams create a square optical lattice. The projection of this lattice on a line at an angle $\alpha = \arctan(2/(\sqrt{5} + 1))$ relative to the lattice creates a quasi-periodic sequence of bond lengths, and hence of hopping energies, which obey the Fibonacci rule. The atoms can be made to travel along the sequence by pointing along this direction a hollow beam (for a description of the latter see for example the work of Xu *et al.* [8]).

As for the constant external drive, a simple way to create and control it would be to tilt the whole set-up by an angle β relative to the vertical axis (see Fig. 4). In this case the external force acting on the condensate atoms is $F = mg \cos \beta$.

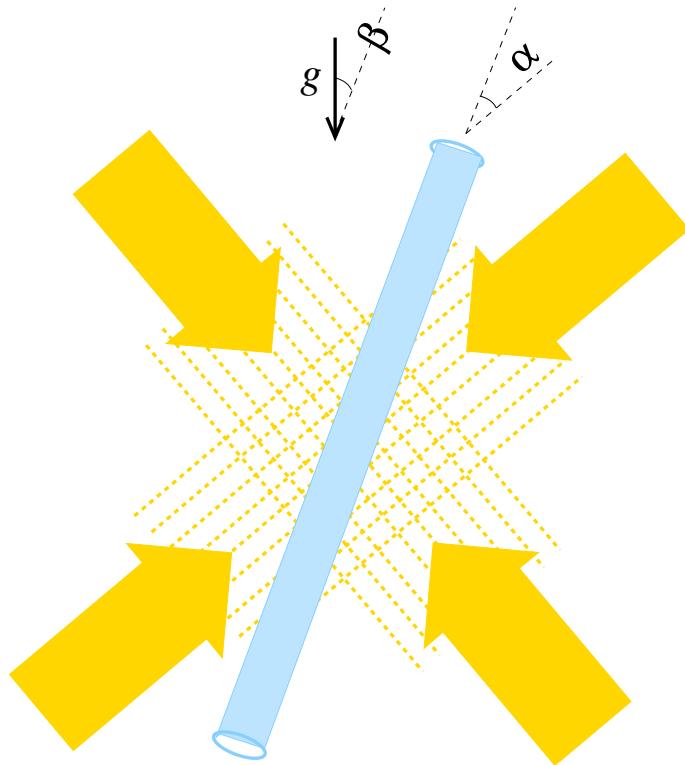


Fig. 4. Schematic representation of a five laser-beam configuration to create a quasi-one-dimensional Fibonacci array of potential wells with quasi-periodic hopping energy. Four beams generate a square optical lattice and a hollow beam confines the condensate to a strip with slope $\alpha = \arctan(2/(\sqrt{5}+1))$ relative to an axis of the lattice. The angle β between the hollow beam and the vertical direction determines the driving force as $F = mg \cos \beta$.

6 Conclusions

In summary, we have shown that localization can result in a Bose-Einstein condensate propagating along a quasi-periodic array of potential wells from the opening of sharp depressions ("minigaps") in the spectral density of states due to quasi-periodic disorder in the site energies. A similar situation will arise when the quasi-periodic disorder is generated in the hopping energies for condensate atoms from well to well in an array, and we have proposed a five-laser set-up by which this situation could be created in the laboratory.

It also seems worthwhile to recall that in Ref. [2] we have interpreted the structure in the transmittivity coefficient as being the result of interference between matter waves propagating through the quasi-periodic array. This interpretation was based on the qualitative affinity between the minigaps in the DOS of the quasi-periodic array in Fig. 1 and the minigap that can be created at the centre of the band in a periodic system by doubling its periodicity. The doubled-period set-up has an optical analogue in an apparatus that performs beam splitting followed by beam interference, and leads to a (regular) structure of maxima and minima in the transmittivity coefficient as a function of a constant drive.

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